STAT 2593

Lecture 032 - Tests Regarding a Population Proportion

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Tests Regarding a Population Proportion

Learning Objectives

1. Understand how we test population proportions.

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- How do we find our p-value?

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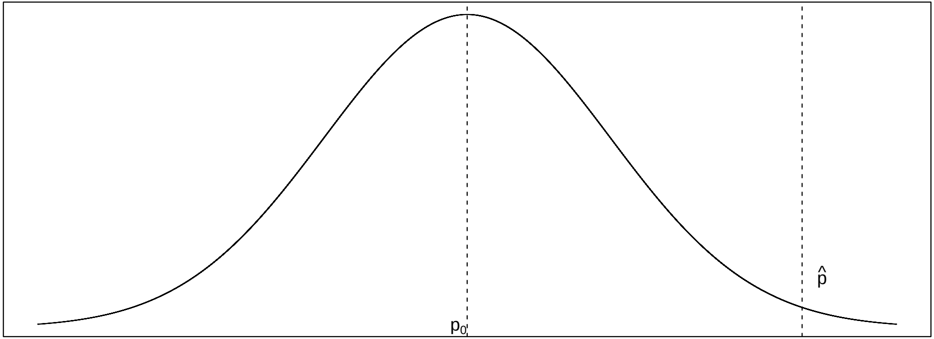

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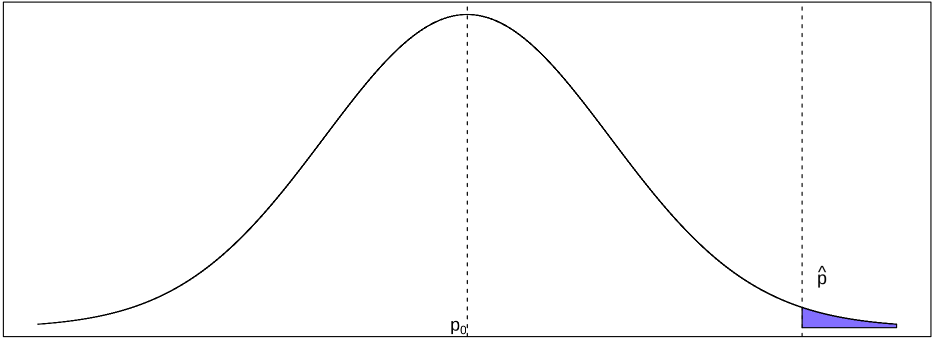

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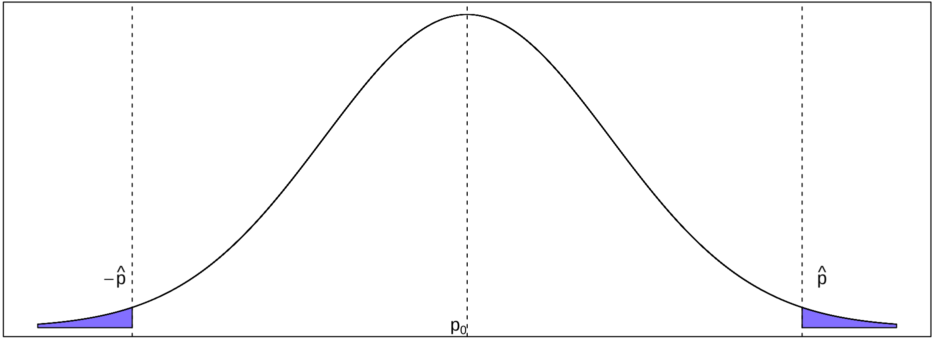

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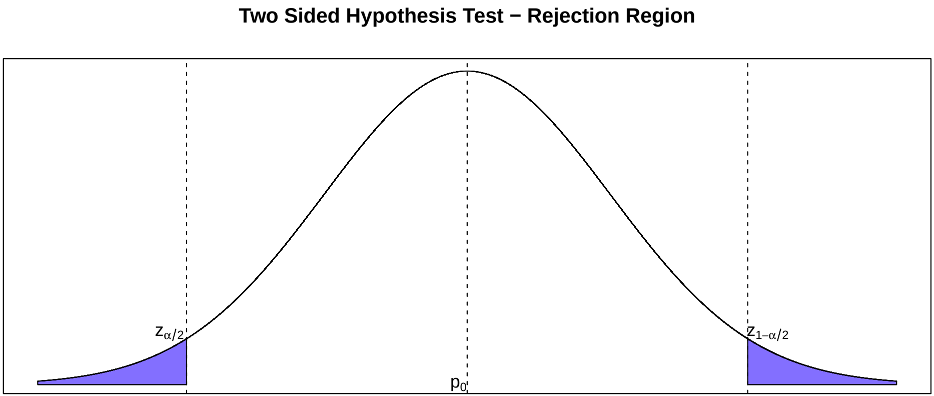
  - Note here we do not take the absolute value.

Rejection Regions for Hypothesis Tests - Critical Values



Summary

- When the normal approximation to the binomial distribution applies, can use a $N(0, 1)$ to run hypothesis tests.
- The rejection region depends on the alternative being considered.